Mathematical Modelling of Problems of Control Theory, Elasticity Theory, Hydro-Thermodynamics, and Statistics Connected with the Terrorist Attacks and Defence against Terrorism

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Unfortunately, terrorist attacks became very frequent recently. Terrorism is an unacceptable form of expression of fight and protest, since in the result mainly die and suffer absolutely innocent population. The number of the countries, where terrorist attacks have had place in the last decade, is very large. Unfortunately, Georgia is among them (terrorist attacks on gas and oil pipelines, thermo- and hydro power plants, power transmission lines, railways carrying the oil, Georgian President, local and foreign citizens and so on). The fight against terrorism, development of the defensive mechanisms against terrorist attacks and prognosis of the possible damage and pollution of the environment as a result of these attacks became a prior problem of the modern science. Mathematical modelling represents a quite convenient and powerful mean to investigate the defence policies against terrorist attacks, as well as the possible results of the terrorist attack. Here we present the list of the themes, connected with the defence against terrorism, which have been developed and investigated in the I.Vekua Institute of Applied Mathematics of the Tbilisi State University:

1.0 CRYPTOGRAPHIC MANAGEMENT OF FLYING OBJECTS FROM THE EARTH SURFACE UNDER THE STRONG REQUIREMENT OF NOT CHANGING THE ROUTE

There is offered the management of flying objects (airplanes) from the earth surface according to pre-defined path. Neither pilots nor the earth flying control organization dispatchers will be able to change the fixed course. Information distortion and its cryptographic decoding in case of intervention in the communication path connecting the earth with airplane is excluded. This result is obtained as there are used in the complex way: multilevel (hierarchical) coding, data compression and available dynamic (variable in time) cryptography.

The essence of the complex method consists in the following:

1. The multiplayer coding net which is geometrical reflection of the multilevel coding layers. Each layer (fig. 1) of the coding net has its own purpose. Each of them selects essential and important that they must be functionally. There is established structural net (convergence degree) as well as information (compression) intercommunication among the layers. There is established the scale invariance between the layers.
2. Aircraft deviation from the course is simultaneously fixes on all the layers. The permissible amplitude of the off-course is given at first time and is controlled from the earth surface (automatically).
3. The sequence of code fragments, which fixes the course of the aircraft (trajectory), is difficult to decode cryptographically. First of all it is conditioned by the next factors:
   a) The code sequence reflecting the course at the same time represents a cryptogram (coded information) as well as a key to a code itself.

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b) The multilevel coding gives a possibility of changing the representation of the net constantly in time (second, minute and so on). As a result of this the kind of the coding sequence cryptographically changes.

In spite of this the required course is being decoding singly in the input devices. This property lies organically in the multilevel coding [1,2], which is able to perceive form wholly (Gestalt elements).

4. It is possible to bring (to compress) the coding nets to one layer.

5. It is not difficult to understand that we shall deal with a unique navigation system.

In different countries of the world (USA, European countries and so on) tried to create multilevel net system. But its realization is based on the means of program. The basis of the system, offered by us, is the multilevel "artificial eye" coding [1,2], which is more effective, multifunctional and has the wide spectrum of explosion.

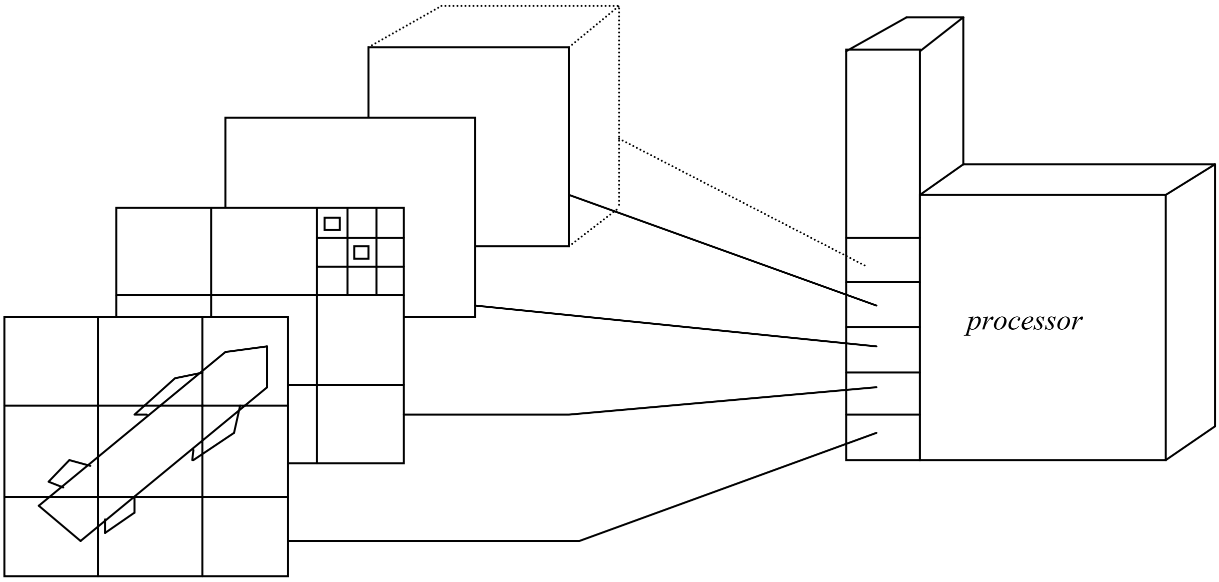

Figure 1.

2.0 MATHEMATICAL MODELLING OF DELOCALISATION AND LOCALIZATION PROBLEMS OF STRESSES AND CONTACT INTERACTIONS OF CONSTRUCTIONS (BODY ARMOURS, SHIP AND TANK STRUCTURES, DEFENSIVE AND CIVIL BUILDINGS), IN VIEW OF THE DEVELOPMENT OF THE DEFENSIVE MECHANISMS AGAINST TERRORIST ATTACKS

In case of local influence on some bodies, sometimes it is important to give them the features, which will allow delocalising the stress inside the body. Examples of such constructions can be: bulletproof vests, turret of tank, defensive structures and so on. On the other hand, on erecting the large-scale constructions, such as large helicopters, multi-storey industrial and residential buildings and etc., it may be necessary to construct them in such a way that to localize more inside the building the outside, though local influence. It is supposed that it will result in the narrow hole in the structure, but not its full destruction and breakdown. Terrorist attacks in 11 September in New York verify the actuality of this problem. Thus, the investigation of the problems connected with the delocalisation and localization of stresses in the elastic bodies, and also their contact interaction, can assist to surviving of people in case of terrorist attacks. In addition, localization of stresses can be used also for striking the fortified objects of terrorists, particularly underground bunkers.

Conventional materials used in construction, mechanic engineering and high-tech industries seldom have the desired combination of properties that would exactly meet specific requirements such as safety, for example. There is empiric evidence which shows that through combining various materials we can often obtain a favorable combination of necessary properties. Thus development of methods for the effective solution of problems of mechanics, which provide an instrument for the optimal use of construction materials, is particularly important.

We suggest solution methods for problems related to stress localization and delocalization in structures affected by surface disturbances and thermal fields. Elasticity problems stated as above have not been considered yet.

Stress localization and delocalization imply redistribution of stresses in a body (medium) when the properties of the medium vary under constant external disturbances and with a basically constant shape. When a body is locally affected by disturbances the deformations in the area of local load are maximal, while outside this area the deformations remain sufficiently small and are almost uniformly distributed in the body. The effective solutions of boundary value contact problems of therm obtained by us provide for a sufficiently uniform distribution of deformations through an appropriate selection of elastic properties of the body, constructive modification of the form of transversal isotropy, introduction of non-homogeneity and partial change of shape. To illustrate a delocalization problem we consider the elastic equilibrium of a free-supported three-layer plate rectangular in the plan, with a local load applied to the upper base. Each isotropic layer has its elastic parameters. The thickness of the layers and contact conditions vary. The solution of the delocalization problem has been obtained for this simple case.

The localization problem for an elastic body is, in a certain sense, inverse to the delocalization problem. The localization problem is defined as follows: to transform the state of a fairly uniform strain in a body into a state of pronounced non-uniform strain (with constant surface disturbances) through an appropriate selection of the parameters of the medium.

The urgent character of the subject is demonstrated by the fact that the obtained results can be widely used in civil and mechanic engineering, high-tech industries, the results being particularly applicable to the cases of point forces, i.e. when the stresses are concentrated. The above-mentioned situations arise in the construction of bridges, high-rise buildings, dams, embankments, in aircraft engineering.

In contrast to optimal design problems (based on particular qualitative optimality properties), the effective solutions of boundary value contact problems of thermo-elasticity obtained by the authors of the given project for transversally isotropic non-homogeneous bodies by a suitable selection of non-homogeneity and transversal isotropy parameters enable one to obtain the desired redistribution of stresses or, to be more precise, to obtain either a uniform distribution of stresses or, vice versa, to reach a pronounced non-uniform stress deformation.

The solution mentioned problems are based on the exact solutions of a whole class of boundary value and boundary value contact problems of elasticity obtained by the Authors [3-5]. The elastic body as assumed to be transversally isotropic, non-homogeneous and multi-layer. The solutions are obtained for bodies bounded by coordinate surfaces of generalized cylindrical coordinates. They include rectangular parallelepipeds, full spheres or their part, elliptical or parabolic plates, etc. Despite the different shapes the mathematical form of the solution remains the same. The efficiency of the solution can be characterized as follows: it in a domain occupied by the body it is possible to construct effective solutions of basic boundary value problems for the Laplace equation, then in the same domain using the same technique the elastic equilibrium of the bodies under consideration can be found as effectively as above. Having exact solutions for a sufficiently wide class of boundary value contact problems one can effectively investigate into problems of localization and delocalization of stresses.

In the commonly used orthogonal curvilinear coordinates (Cartesian, Cylindrical, spherical, cylindrical parabolic, cylindrical elliptic and other systems of coordinates) a thermo-elastic dynamic equilibrium problem for a multi-layer finite or infinite body bounded by the coordinate surfaces of the above coordinates is considered and its analytically effective solution is given. Each layer is a transversal isotropic elastic medium non-homogeneous, along its thickness. Some examples of such multi-layer bodies are given in Fig. 2 and 3.

For simplicity we consider the case of Cartesian coordinates. Further on we state the problem for a static case. As a result we will have a three-dimensional static boundary value contact problem of thermo-elasticity, which is stated below.

Let a multilayer elastic body in a Cartesian system of coordinates x, y, z occupy the domain $\Omega=\{-a<x<a,\ -b<y<b,\ -<z<\infty\}$ (evidently, $0<z<c$ where c = const so that for z = c any homogeneous and non-homogeneous boundary conditions and thermal disturbances cam be considered), which consists of h_1, h_2,... k_k transversally isotropic layers non-homogeneous along z with the thickness (see Fig. 2) with each layer having its own elastic and thermal characteristics. Moreover, for $x = \pm a$ and $y = \pm b$ homogeneous boundary conditions are defined that are natural for a geological massif. Such natural conditions, we believe, are boundary and thermal conditions, which guarantee a continuous extension of the solution. Contact conditions between the layers can be arbitrary; moreover, the disturbances in the multilayer medium can be generated by the nonhomogeneity of those conditions. On the surface $z = 0$ the disturbances can be caused by defining the stresses, displacements or their combinations (thermal disturbances can, naturally, be present in all the three cases).

For the static problems stated above N.Khomasuridze found effective analytical solutions [3-5].

As an example the thermoelastic equilibrium of a rectangular parallelepiped $\Omega\{x_0 < x < x_1, y_0 < y < y_1, z_0 < z < z_1\}$ is considered.

On each lateral surface of the rectangular parallelepiped $x = x_i$ and $y = y_i$ $(i = 0,1)$ symmetry or antisymmetry conditions are defined. On the upper and the lower sides of the rectangular parallelepiped $z = z_i$ disturbances are acting.

Thermoelastic equilibrium of a homogeneous isotropic body is described by the equation

$$\Delta T = \frac{1}{\lambda} T^*, \tag{2.1}$$

$$grad\left(\frac{\kappa\mu}{\kappa - 2} div\bar{u}\right) - rot(\mu rot\bar{u}) = 0, \tag{2.2}$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, λ is the coefficient of thermal conductivity, $\mu = \frac{E}{2(1+\nu)}$, E is the modulus of elasticity, ν is the Poisson ratio, $\kappa = 4(1-\nu)$, $T^* = T^*(x,y,z)$ is the thermal source, $\vec{u} = u\vec{j}_x + v\vec{j}_y + w\vec{j}_z$ is the displacement vector and $\vec{j}_x, \vec{j}_y, \vec{j}_z$ are basic vectors in Cartesian coordinate system, $T = T(x,y,z)$ is the body temperature variation.

The displacement vector components in (2.2) u, v and w are expressed by means of the harmonic functions

$$u = \frac{\partial \varphi_1}{\partial z} - z\frac{\partial \varphi_3}{\partial x}, \qquad v = \frac{\partial \varphi_2}{\partial z} - z\frac{\partial \varphi_3}{\partial y},$$
$$u = (\kappa - 1)\varphi_3 - z\frac{\partial \varphi_3}{\partial z} - \frac{\partial \varphi_1}{\partial x} - \frac{\partial \varphi_3}{\partial y}. \tag{2.3}$$

The harmonic functions φ_1, φ_2 and φ_3 in (2.3) can be represented as [3-5]:

$$\varphi_3 = a_{30} + a_{31}z$$
$$+ \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\left\{A_{mn}\frac{\sinh(pz)}{\sinh(pz_1)} + B_{mn}\frac{\sinh(p(z-z_1))}{\sinh(pz_1)}\right\}\cos\left(\frac{\pi m}{x_1}x\right)\cos\left(\frac{\pi n}{y_1}y\right),$$
$$\varphi_1 = \sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\left\{C_{mn}\frac{\sinh(pz)}{\sinh(pz_1)} + E_{mn}\frac{\sinh(p(z-z_1))}{\sinh(pz_1)}\right\}\sin\left(\frac{\pi m}{x_1}x\right)\cos\left(\frac{\pi n}{y_1}y\right),$$
$$\varphi_2 = \sum_{m=0}^{\infty}\sum_{n=1}^{\infty}\left\{G_{mn}\frac{\sinh(pz)}{\sinh(pz_1)} + H_{mn}\frac{\sinh(p(z-z_1))}{\sinh(pz_1)}\right\}\cos\left(\frac{\pi m}{x_1}x\right)\sin\left(\frac{\pi n}{y_1}y\right),$$

where $p = \sqrt{\left(\frac{\pi m}{x_1}\right)^2 + \left(\frac{\pi m}{y_1}\right)^2}$; $A_{mn}, B_{mn}, \ldots, H_{mn}$ are constants.

The constants $A_{mn}, B_{mn}, \ldots, H_{mn}$ can be easily defined as solutions of the corresponding system of algebraic equations. This technique can be extended to analytical solutions of the corresponding dynamic problems.

With the above-mentioned analytical solutions at our disposal we can solve the following problems:

I. For constant surface and thermal disturbances given on z = 0 (including deep disturbances which can be generated by rtonhomogeneous contact conditions), uniformly distributed (the distribution could be arbitrary as well) select elastic and thermal characteristics of the Layers, conflict conditions, thickness of the layers.. h_1, h_2,… k_k, their number k and sequence, so that *in* the neighborhood of point M (see Fig. 2) situated at the distance l of the plane we could localize (concentrate) the stresses.

II. For constant surface and thermal disturbances defined on z = 0 (including deep disturbances induced by nonhomogeneous contact conditions) which are, for instance, locally distributed, or, in other words, are concentrated (the distribution can be arbitrary as well) select (be parameters listed above in T. so that on the plane of the rectangle z = l we could obtain a uniform distribution of the parameters, i.e. delocalize the stresses.

III. Under constant geometry and physics Of the elastic multilayer body (i.e. when the number of layers, their thickness, sequence and their elastic and thermal characteristics are constant) select surface and thermal disturbances on z = 0 (including (he selection of deep disturbances induced by the nonhomogeneily of contact conditions), so that in the neighbourhood of point M we could concentrate or localize the stresses.

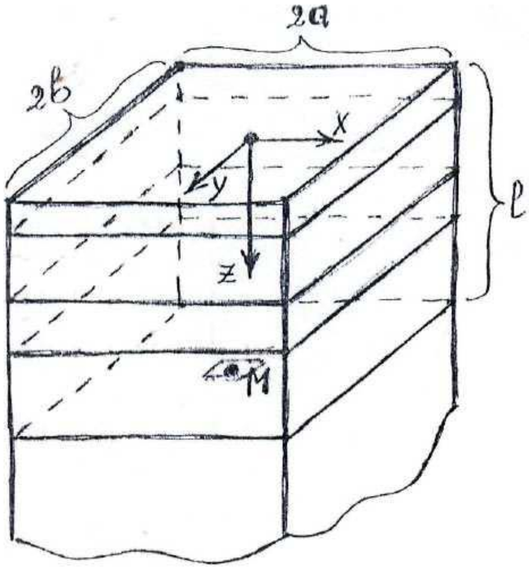

Figure 2.

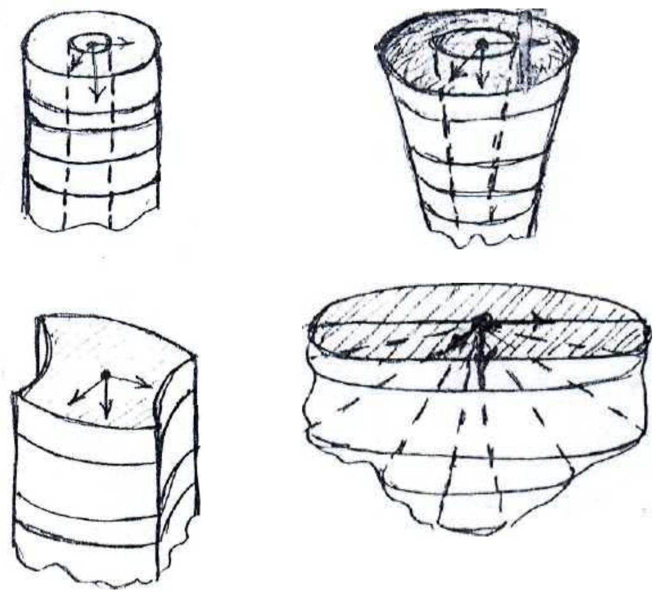

Figure 3.

IV. Under constant geometry and physics of the elastic multilayer body select surface and thermal disturbances On z = 0 (including the selection of deep disturbances induced by the nonhomogeneity of contact conditions), so that on the plane of the rectangle z = l we could have a uniform distribution of the stresses, i.e. we could delocalize the stresses.

3.0 SOFTWARE FOR OPERATIONAL DETECTION OF THE NON-ORDINARY POLLUTANTS OF RIVERS (COMPATIBLE TO IBM P/C). AUTOMATED WATER QUALITY CONTROL SYSTEM CONNECTED WITH TERRORIST ATTACKS

Among the terrorist actions, related to mass empoisoning and destruction, is mostly dangerous to empoison the portable water by the chemical or biological weapons. This act can take place in the big industrial cities as well as in the moderate or the small-inhabited localities. If in the big cities the portable water service has expensive high technologies water quality test equipments, but we can't say the same about small communities and the villages. Terrorist actions can take place in the big industrial cities, for example: when they use the masked poisoning substance of late-action, which reacts with the portable water cleaning substances. In any case, the monitoring and control of the rivers, which are used for portable water in the neighborhood of the portable water tanks (before plumping into the tank as well as after it) it is reasonable and necessary to implement the automatic monitoring in the water delivery system in every inhabited localities. By the other side, it is also necessary monitoring and control of the artificial and natural water storages.

One of the main sections of stochastic mathematics is decision theory. There are different methods of statistical hypotheses testing, depending on the criterion chosen. Among them, a special place has Bayesian approach, which allows more complete use of a priori information and making of guaranteed decisions. The Optimal Decision-Making Application Package is intended for solution of a particular case of the decision-making problem, namely, for the problem of making stochastic decisions on the basis of experimental results by means of one of the classical approaches – the Bayesian approach. The mathematical basis for decision-making in recognition problems is theory of games and, especially, one of its specific branches – statistical decision that can be considered as optimal decision-making theory on the basis of carrying out tests.

In spite of the variety of papers devoted to the problem of statistical hypotheses testing and, in particular, to Bayesian criterion of hypotheses testing, none of these papers, however, considers Bayesian problem of many hypotheses testing from that point of view as it is considered in the presented package. Here, besides the unconditional optimization problem of mean risk minimization, the conditional optimization problem is solved that allows, by imposition restrictions on the first kind error probability, to minimize the error of second kind. Besides, the package implements simple, convenient and reliable decision-making algorithms based on the cluster-analysis method.

The package was developed as a convenient, reliable, simple in use tool for optimal decision-making on the basis of experimental data for considering practical problems in various branches of science and practice. The package is developed for IBM-compatible personal computers in operation system WINDOWS.

The work is of great theoretical importance, as it is a new, significant contribution to the theory of statistical hypotheses testing. It is of great practical importance as well, as many problems in technology, ecology, medicine, sociology, economics, physics, chemistry and other branches come to the scheme of statistical hypotheses acceptance, and to have a simple, convenient, comprehensible tool that can be easily used by any subject specialist who has a little knowledge of mathematics and computer engineering is highly important.

The developed package is original, has no analogue. Two kinds of approaches to the Bayesian hypotheses-testing problem are used in it: the classical one that consists of the finding of the decision rule that minimizes the risk function containing both kinds of errors; a new approach that consists in finding the decision rule by solving risk function conditional optimization problem minimizing the total probability of true decisions rejection, at restrictions on the probability of making wrong decisions concerning the truth of hypothetical values of distribution parameters. The package implements both optimal decision-making algorithms and quasi-optimal hypotheses testing rules that allow make decisions without considerable computer time losses. The package also implements decision-making algorithms based on the cluster-analysis method, in particular, based on Euclidean and Makhalanobis distances.

Automated Water Quality Control System

The automated water quality control system operates within data processing center. It is realized on the basis of IBM compatible computer with the use of commuted dropped off communication lines and gets measurement information on the concentration of parameters monitored from the automated water quality analyzers via the data transmission unit of MODEM type. Information software as well as mathematical software is realized as a system of application programs. The automated water quality control system functions within the MS DOS operating system and can include very much (practically any number) of analyzers, each of them measuring up to 20 various ingredients.

The main features of the system are: operative processing of measurement information, validity of the results obtained, rapid adaptation to changes of controlled objects parameters without human intervention, simplicity of maintenance, flexibility when using the system for monitoring and analysis, visual presentation of the results obtained.

The information software realizes the following tasks:

- Automatic interrogation of water quality analyzers included into the system;
- Reception, deciphering and recording of the measurement information coming from the automated analyzers onto the carriers;

- Control of analyzers operating modes (start of the analyzer, inquiry for an additional measurement, change in step of analyzers operation, arbitrary sampling, inquiry for information recovery after the troubleshooting in case there has been a break in analyzers connection to the data processing center);
- Display of the current and retrospective measurements on the video terminal screen, support of communication between the operator at data processing center and data base (generation, updating and liquidation of system's directory and files), control of timely reception of measurement results from the analyzers, information output in accepted forms according to the user's order (24-hourly printout per analyzer; printing of master reports per analyzer for a day, a decade, a month; printing of reports on the violation of hydrological regimes), data archiving, processing of information coming from sanitary or mobile hydro chemical laboratories as well as stationary hydro chemical laboratories within surface water control system and printing of reports for 24-hours, a decade, a month, a quarter and a year; display of information on the video terminal by dispatcher's order.

A special attention is paid to displaying of information which includes graphical displaying of change in parameters controlled with time, visual interpretation of the states of water body controlled and of water quality control system, statistic processing of measurement results per each parameter and presentation of results of this processing in easy-to-use form, etc.

The mathematical software of the system realizes the following tasks:

- Preliminary statistic processing of the measurement results (calculation of the minimum, maximum and mean values of the ingredients measured in a given period of time);
- Calculation of trends for the temporal series;
- Calculation of automatic and intercorrelation functions for the temporal series;
- Classification of the state of the tests media in a discrete moment of time and evaluation of adequacy of the control (printing of alarm warnings; alarms in case of natural environment contamination);
- Calculation of the predicted values of the ingredients controlled with a given lead value;
- Classification according to the predicted values and calculation of adequacy of the decisions mode;
- Detection of trends of change in the state of media controlled; selection of optimum step for the discrete control of water quality; analysis of the ingredients measured for "anomaly" (i.e. abrupt change in the concentration of given ingredient);
- Search for a contaminer; Calculation of the maximum permissible discharge.

Communication with the information software and use of the information accumulated in the system for drawing up reports, carrying out hydrological, hydro biological, hydro chemical and other studies by methods of applied mathematics as well as visual presentation of the results obtained is carried out on the basis of the given menu on the computer display: system adjustment, operation with a database, reports, research, forecasting, mobile hydro chemical laboratories. In each menu respective submenus are realized. In particular, system adjustment – start, commands, initialization; operation with the database – generation of a file directory, file generation, file preparation, file nullification, communication with the information base, file annulment, maintenance of standardized information; reports-information display, operational paper, 24-hour report, master report over a decade, master report over a month, master report over a quarter of the year, master report over six months, annual master report; research – plots of the parameters measured, statistical study of the parameters measured, regression analysis of the parameters measured, correlation analysis of the parameters measured, forecasting of the values for the parameter measured; forecasting – routine, short-term I, short-term II, comparison. The following possibilities are realized in the submenu "commands": change in the measurement step, extra measurement, arbitrary sampling, regeneration of the information; "Generation of the file directory" offers the following: formation, abridgment, broadening and annulment of the directory. "Regression analysis" enables the regeneration of auto- and interregression between the parameters measured. Results of the each task are presented in the form of a text, table or plot, which can be output to the computer monitor, printer or plotter, if any.

Mathematical Models Describing Pollutant Transfer in Water Courses

As is known, the partrial differential equation, which most fully describes pollutant transfer in water courses, has following form [6,7]:

$$\frac{\partial S}{\partial t} = \frac{\partial}{\partial x}\left(k, \frac{\partial S}{\partial x}\right) + \frac{\partial}{\partial y}\left(k, \frac{\partial S}{\partial y}\right) + \frac{\partial}{\partial z}\left(k, \frac{\partial S}{\partial z}\right) \tag{3.1}$$

$$- v_x \frac{\partial S}{\partial x} - v_y \frac{\partial S}{\partial y} - v_z \frac{\partial S}{\partial z} - u\frac{\partial S}{\partial y} - kS + F,$$

where x, y, z – spatial coordinates, t – time; S ≡ S(t, x, y, z) – concentration of the pollutant, kx ≡ kx (t, x, y, z), ky ≡ ky (t, x, y, z), kz ≡ kz (t, x, y, z), – coefficients of turbulent diffusion in the direction of axes Ox, Oy, Oz; vx ≡ vx (t, x, y, z), vy ≡ vy (t,x,y,z), vz ≡ vz (t, x, y, z), – velocity vector components along coordinate axes Ox, Oy, Oz; k – nonconservativity coefficient; u – hydraulic size of particles; F ≡ F(t, x, y, z) – power of external sources.

Equation (3.1) together with the initial condition and boundary conditions represents the mathematical model of the process in question. To specify the initial and boundary conditions, let's define the region, occupied by the water course. Let's assume that, in space of points M ≡ (x, y, z) (is taken the orthogonal system of coordinates), water course occupiers region G with boundary Γ , Γ = Γ1 ∪Γ2 ∪ Γ3 ∪ Γ4 ∪ Γ5 ∪ Γ6 ; Γ1 – the upper range, Γ2 and Γ 4 – side walls, Γ5 – bottom, Γ6 –free surface, Γ3 – the lower range . Let Γ0 denote the cross-section of water course between the upper and the lower ranges.

To define the concentration in G within time interval [0,T] (T = const) for equation (1), the following conditions are specified:

$$S(0, x, y, z) = S_0(0, x, y, z), \qquad (x, y, z) \in G, \tag{3.2}$$

boundary conditions

$$S(t, x, y, z) = \phi_1(t, x, y, z), \quad (x, y, z) \in \Gamma_1, \quad t \in [0,T] \tag{3.3}$$

$$\frac{\partial S}{\partial v} + \chi S = \varphi_2 \quad (t,x,y,z), \quad (x,y,z) \in \Gamma_2 \cup \Gamma_4 \cup \Gamma_5 \cup \Gamma_6, t \in [0,T] \tag{3.4}$$

$$\frac{\partial S}{\partial v} = 0, \qquad (x,y,z) \in \Gamma_3, \qquad t \in [0,T] \tag{3.5}$$

where S 0(x, y, z), ϕ_1 (t, x, y, z), ϕ_2 (t, x, y, z), $\chi \equiv \chi(t,x,y,z)$, – given functions, v – outward normal to Γ.

Mathematical model (3.1) – (3.5) is the traditional statement of such type problems; it represents the classical problem of mathematical physics and its correctness may be shown under some assumption of quite a general form.

In some cases condition (3.5) may be changed by condition

$$S(t, x, y, z) = q(t, x, y, z)S(t, x^*, y^*, z^*) + \varphi_3(t, x, y, z) \qquad (3.6)$$

$$(x, y, z) \in \Gamma_3, \quad (x^*, y^*, z^*) \in \Gamma_0, \qquad 0 \le t \le T,$$

where q – coefficient of self-purification $(0 \le q \le 1)$, – given function and $\Gamma_0 = I(\Gamma_3)$ $I(.)$ – diffeomorphism between Γ_0 and Γ_3 .

Condition (3.6) is specified in mathematical modelling problems of pollution transfer for the first time, and, obviously, it has quite a definite practical value.

It may be proved that problem (3.1) – (3.4), (3.6) is stated correctly.

To receive from (3.1) – (3.5) or (3.1) – (3.4), (3.6) one – or two-dimensional averaged models or, in general, more simple models, different assumptions of physical, mechanical or geometrical character are made [6,7].

Realization of full three-dimensional problem on computers is associated with receiving of quite an expensive detailed information of high accuracy and with great computational difficulties.

For practical calculations, two-and one-dimensional mathematical models (see [6,7]) are often used with due regard for the specificity of the studied water courses.

4.0 MATHEMATICAL MODELLING OF THE DAMAGE OF HYDROPOWER PLANTS AS A RESULT OF WRECK EVENTS IN THE MOUNTAINS

In nowadays the construction and exploitation of hydroelectric power stations with regulating reservoirs is of great importance for every country. Natural and handmade reservoirs represent potential danger to catastrophic events (the consequence of different kinds of landslide events causes formation of gravitational waves; dam may be collapsed by natural (seismic) or handmade (sabotage, act of terrorism) events). In mountainous countries, mentioned reservoirs are located mainly in mountainous and semi-mountainous regions. Such reservoirs are characterized with small mirror surface and sharp level volatility. Besides, influence of filtration, ground waters and atmospheric precipitations weakens internal resistance of rocks, what leads to development of deep (scaled) creepage. Moreover, if the reservoir is located in seismic region possibility of development of landslide events is significantly increased.

Natural and handmade reservoirs represent potential danger to catastrophic events. Particularly, on water surface at reservoir board in consequence of different kinds of landslide events, gravitational waves are originated. For example, in France in 1959, as a result of rocks subsidence, basement of Malpasset dam was destructed, what caused its collapse and death of more then 400 people; in 1963 in consequence of 250 million m^3 landslide massif creeping, overtopping of 70-meters height water wave over dam caused death of thousands of people in Vaiont reservoir in Italy. In India, in 1979 the dam collapsed because of insufficient capacity, 2 thousand people were dead. In October 2000 from iced peak of mountain Altar located in the center of Ecuador, soaked stone block of approximately 1.5. million m^3 volume, stone mass rumbled downstream and collapsed in icy lake Amarilia. As a result of powerful impact, inflicted from approximately 900 meters of height, originated gravitational waves covered whole floor of the valley with the dirt layer of thickness of approximately 20 meters. According to existing data, from 1800 to 1983, 60 emergency cases of dam collapse causing death of 16 000 people were fixed.

In order to evacuate population and valuables located downstream in time when dam collapses or when waves are formed in consequence of different kinds of landslide events, clarity of prediction of water masses propagation characteristics – motion velocity, front shape, expansion surface etc. is vivid. These makes actual to accurate, implement and establish mathematical models of such parameters calculation. Presented work is dedicated to research of mentioned concrete problems.

Mathematical model of wave formation and propagation can be constructed on the background of common hydrodynamic equations with boundary conditions considering presence of water free surface. However, straight usage of mentioned equations for concrete problems is related to extreme complexities of mathematical and technical character. Thus, in concrete researches, are used theoretical models of fluid motion received on the base of those simplifying presumptions, which allow finding of numerical results relative to different aspects of fluid behavior.

Two-Dimensional Mathematical Model of Breaking Wave Formation and Propagation when Dam Collapses

In this chapter two-dimensional mathematical problem of adjacent dam collapse will be solved and received numerical results approximated with the use of NURBS surfaces [8-11].

Two dams are called adjacent, if they are located in canyons outgoing to the same plane. Two-dimensional mathematical problem is stated as follows: at the initial moment of time, dams located at the upper right and lower left corners of computational area are instantly collapsed, thus, two powerful flows moving towards each other are formed. Computational domain of a given problem is square $D = \{0 \le x \le a, 0 \le y \le a\}$.

The basis of a mathematical model of breaking wave formation and propagation when dam collapses is the assumption, that vertical component of water particles acceleration slightly affects the pressure, or in other words pressure distribution in fluid obeys the hydrostatic low (shallow water theory). Considering the water as an ideal fluid and the water stream potential, water flow can be described by Saint-Venant equations. In addition it is considered, that water is of constant depth as in upper so in lower pool of dam.

At the initial moment of time motion of breaking waves formed when dam collapses, is characterized with sharp water level changing on short section and with curved profile of free surface. Consequent propagation of breaking wave causes comparably small changes of levels in lower pool.

If for any time moment as for unknown variables are considered depth of riverbed, or water level, and for given cutting plane (averaged) velocity distribution, water motion into channel or in open riverbed can be described by Saint-Venant system of differential equations.

Let us write corresponding two dimensional mathematical model in divergence form for unknown impulse's $J_1(x,y,t)$ and $J_2(x,y,t)$ components and $H(x,y,t)$ flow depth. In vector form it has following form[8-11]:

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} F_1(Q) + \frac{\partial}{\partial y} F_2(Q) = W(Q), \qquad (4.1)$$

where, $J_1 = Hu$, $J_2 = H\upsilon$, $Q = (J_1, J_2, H)^T$,

$$F_1(Q) = \left(J_1 u + gH^2/2,\, J_1\upsilon,\, J_1\right)^T,$$

$$F_2(Q) = \left(J_2 u,\, J_2\upsilon + gH^2/2,\, J_2\right)^T,$$

$$W(Q) = \left(f_1(x,y,t,H,u,\upsilon), f_2(x,y,t,H,u,\upsilon), f_3(x,y,t,H,u,\upsilon)\right)^T,$$

$$H(x,y,0)=\begin{cases} a_1 & (x,y)\in D_1\,, \\ a_2 & (x,y)\in D_2\,, \\ a_3 & (x,y)\in D_3\,. \end{cases} \tag{4.2}$$

$$u(x,y,0)=\upsilon(x,y,0)=0, \quad (x,y)\in D=D_1\cup D_2\cup D_3\,, \tag{4.3}$$

x and y are Euler variables, t–is time variable, g-acceleration of gravity, $u(x,y,t)$ and $\upsilon(x,y,t)$ -projections of velocity vector on Ox and Oy axes accordingly. $a_1, a_2, a_3 = const$,
$f_1(x,y,t,H,u,\upsilon), f_2(x,y,t,H,u,\upsilon),\ f_3(x,y,t,H,u,\upsilon)$ – functions, describing sources of mass and impulse.

Numerical Experiments

In order to numerically solve (4.1)-(4.3) two dimensional mathematical model, two layer difference scheme with non-linear regularizing functional $R_i(Q),\ i=1{,}2$, which do not changes order of scheme and has a meaning of "artificial viscosity", will be used [12]

$$Q_{ht}+\left[F_{h1}(Q_h,\sigma)\right]_{\overset{\circ}{x}}+\left[F_{h2}(Q_h,\sigma)\right]_{\overset{\circ}{y}}=R_{h1}(Q_h)+R_{h2}(Q_h)+W_h(Q_h)\,, \tag{4.4}$$

$$\begin{aligned} &H_h(ih,jh,0)=H_0(ih,jh),\\ &u_h(ih,jh,0)=u_0(ih,jh),\ \upsilon_h(ih,jh,0)=\upsilon_0(ih,jh). \end{aligned} \tag{4.5}$$

where, grid functions

$$Q_h=\left(J_{h1},\,J_{h2,}\,H_h\right)^T,\ W_h(Q_h)=\left(f_{h1},f_{h2},f_{h3}\right),$$
$$F_{h1}(Q_h)=\left(J_{h1}u_h+0.5gH_h{}^2,\ J_{h1}\upsilon_h,\ J_{h1}\right)^T,$$
$$F_{h2}(Q)=\left(J_{h2}u_h,\ J_{h2}\upsilon_h+0.5gH_h{}^2,\ J_{h2}\right)^T,$$

where, τ and h are steps in time and space accordingly,

$$r_{h1}(Q_h)=0.25H_h\left((\lambda_{11}+\lambda_{12})u_h{}^{(\sigma)}+0.5c_h(\lambda_{11}-\lambda_{12}),(\lambda_{11}+\lambda_{12})\upsilon_h{}^{(\sigma)},\ (\lambda_{11}+\lambda_{12})\right)^T,$$
$$r_{h1}(Q_h)=0.25H_h\left((\lambda_{21}+\lambda_{22})u_h{}^{(\sigma)},(\lambda_{21}+\lambda_{22})\upsilon_h{}^{(\sigma)}+0.5c_h(\lambda_{21}-\lambda_{22}),\ (\lambda_{21}+\lambda_{22})\right)^T,$$

where , $\lambda_{11}=|u_h+c_h|,\ \lambda_{12}=|u_h-c_h|,\ \lambda_{21}=|\upsilon_h+c_h|,\ \lambda_{22}=|\upsilon_h-c_h|,\ c_h=\sqrt{gH_h}$ – is a difference analogue of velocity 's perturbation propagation, $\sigma=\hat{H}_h{}^{1/2}\Big/\left(\hat{H}_h{}^{1/2}+H_h{}^{1/2}\right)$; $\varphi_k(\tau,h),k=1{,}2$ are sufficiently smooth functions of its variables and $\varphi_k(\tau,h)\to 0,\ k=1{,}2$ when $\tau\to 0, h\to 0$, $\varphi_k(\tau,h)=O(h^2)$; grid functions $H_h=H(ih,ji,k\tau),\ u_h=u(ih,ji,k\tau),\quad \upsilon_h=\upsilon(ih,ji,k\tau),\ f_{lh}=f_l(ih,ji,k\tau),\ l=1{,}2{,}3$ are defined on $D=\varpi_{h^2}\times\varpi_\tau$ area and approximate H,u,υ,f_l , $l=1{,}2{,}3$ functions accordingly. $H_0(ih,jh),\ u_0(ih,jh),\ \upsilon_0(ih,jh)$ are difference analogues of depth and velocity components distribution functions correspondingly.

Problem Statement: Two dams of same height are located in opposite canyons of same valley. At the initial moment of time dams are instantly collapsed and two powerful water flows moving towards each other are formed. To describe dynamic of water flows (4.1) – (4.3) problem is solved with (4.4) – (4.5) difference scheme. As a computational domain is taken $D=\{0\le x\le 10, 0\le y\le 10\}$ square; dams are modeled as planes, projections of which are represented by lines in upper and lower corners of square. Initial conditions are of following form:

$$H(x,y,0)=\begin{cases}10, & (x,y)\in D_1\\ 10, & (x,y)\in D_2\\ 1, & (x,y)\in D\setminus\{D_1\cup D_2\},\end{cases}$$

$$u(x,y,0)=\upsilon(x,y,0)=0,\ (x,y)\in D,$$

where, $D_1=\{(x,y):\ x\in[0,2.5]\}$, $D_2=\{(x,y):\ x\in[7.5,10], y\in[17.5-x,10]\}$.

In boundary conditions presence of solid walls are taken into consideration by zeroth component of velocity vector on corresponding boundary points of considered domain. Computation in both spatial directions are made with step $h=0.1$ in conditions, that $f_1(x,y,t,H,u,\upsilon)=f_2(x,y,t,H,u,\upsilon)=f_3(x,y,t,H,u,\upsilon)=0$. Relation between spatial and time steps is chosen from CFL condition:

$$\tau\le Cr\cdot h\Big/\max_i\{(|u_i|+c),\ (|\upsilon_i|+c)\}$$

where, Cr is a Courant number.

Results of numerical calculations for flow's H depth and u velocity vector as NURBS surfaces are shown on figures 4-7 for time moments:

At the initial time moment (**Fig. 4**) dams are instantly collapsed and two powerful water flows moving towards each other are formed. After collision time moment $t=0.3\,sec.$ (**Fig. 5** *a*) the bigger volume water flow ingests oppositely moving second flow, what causes non-uniform distribution of parameters across the diagonal of the square. Breaking wave propagates towards free corner of the computational area (**Fig. 6** *a*); after reflection (**Fig. 7** *a*), some small time later, it receives a form of "unitary wave" and keeps moving towards opposite corner across the main diagonal of the square. This motion is continued until steady motion is reached.

At collision time moment $t=0.3\,sec.$ (**Fig. 5** *b*) the flows reach their peak velocity, the faster water flow ingests oppositely moving second flow and makes the breaking wave to propagate towards the free corner of the computational area (**Fig. 6** *b*); after reflection (**Fig. 7** *b*), some small time later, the velocity vector reaches its second peak as a velocity of formed "unitary wave", which keeps moving towards opposite corner across the main diagonal of the square.

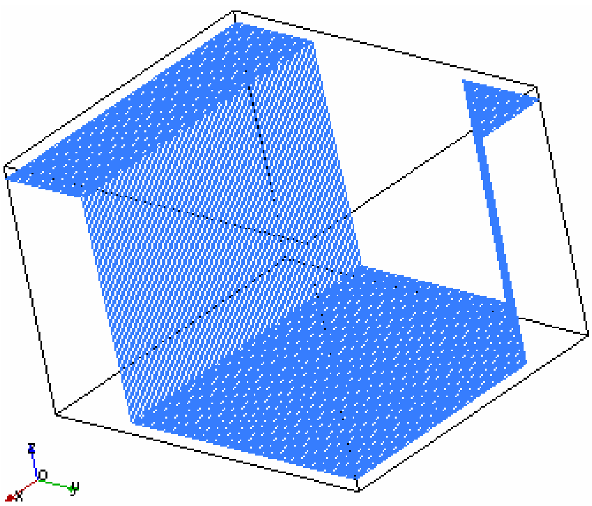

Figure 4: $t=0$ ***sec.***

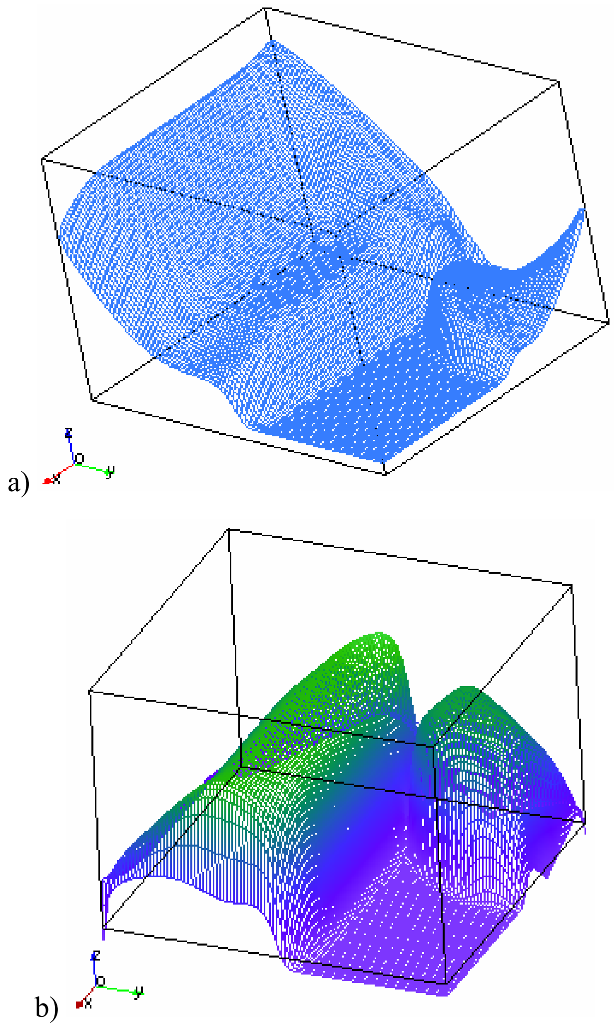

Figure 5: $t = 0.3$ *sec.*

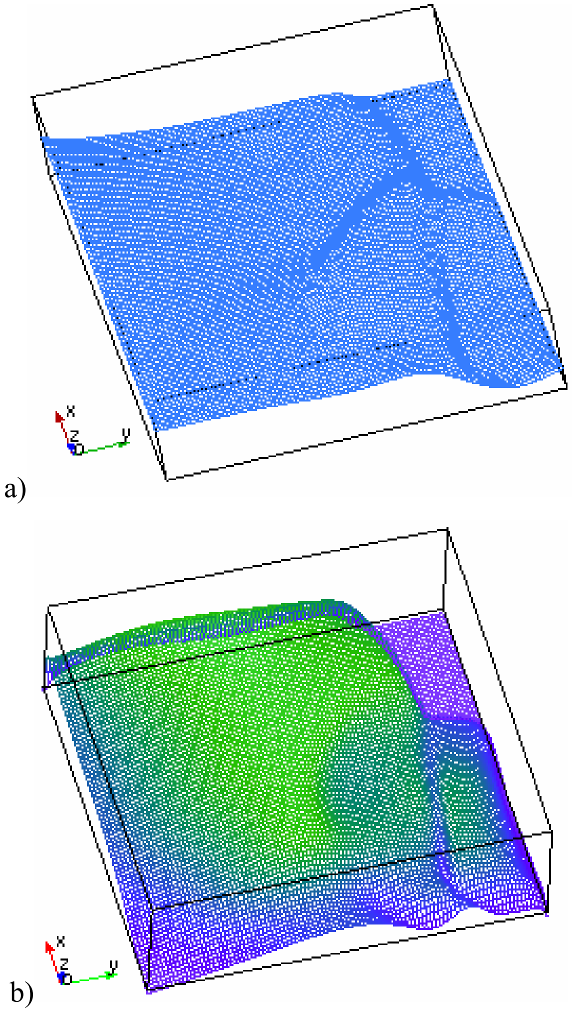

Figure 6: $t = 0.6$ *sec.*

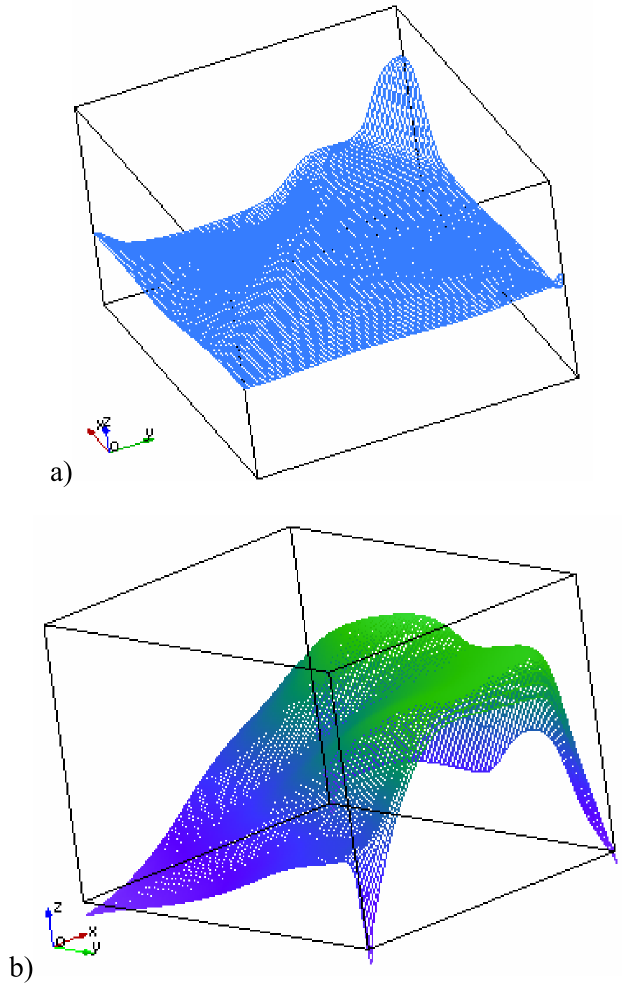

Figure 7: $t = 1.25$ *sec.*

5.0 MATHEMATICAL MODELLING OF THE POLLUTION OF THE SOIL AND UNDERGROUND WATERS AS A RESULT OF EXPLOSIONS OF OIL-PIPELINES, RAILWAYS CARRYING THE OIL, AND OIL TERMINALS

As foreign practice of pipeline exploitation shows, the main reason of crashes and spillages (and fires as a consequence) is destruction of pipes as a result of corrosion, defects of welding, natural phenomena and so on (including terrorist attacks and sabotage). The probability of crashes for pipeline transport rises with the age of the oil pipelines in service, and with the extent of their network.

The oil spillage caused by oil transportation through pipeline and railway results in serious deterioration of soil, underground water and rivers, which is one of the causes of the sea pollution. All these processes have very bad impact on environment, and therefore, on human health. Therefore it is very important to carry out precautionary model studies of possible emergency situations. The main result will be identification and assessment of the ecological impacts resulting from oil spills and its control with substances possible harmful for man and nature.

As known the European Union is one of the main ideologists and sponsors of the transport corridor Europe-Caucasus-Asia (TRACECA). EU considers TRACECA as mechanism of the inter-state and inter-regional collaborations and the guarantee of peace and stability. EU considers Georgia as a partner in the development of the transport networks between the Black Sea and Central Asia because of its geopolitical position.

At present there are already functioning the following routs of oil and gas transportation via the territory of Georgia (The list of possibilities of terrorist attacks and sabotage and oil and oil-products spilling on the territory of Georgia):

- Baku(Azerbaijan)-Supsa(Georgia) pipeline – the Western Export Pipeline route (WEP);
- Tengiz deposit(Kazakhstan)-Baku(Azerbaijan) – Batumi (Georgia)(TBB) route- fulfils a transportation of oil by railway;
- Baku(Azerbaijan)-Poti(Georgia) (BaP) route – fulfils a transportation of oil by railway;
- Baku(Azerbaijan)-Kulevi(Georgia) (BK) route – fulfils a transportation of oil by railway;
- Vladikavkaz(Russia)-Erevan(Armenia) (VE) pipeline route – fulfils a transportation of gas by pipeline from Russia , through Georgia to Erevan.
- Also there are in the building a new pipeline systems:
    - Baku(Azerbaijan)-Tbilisi(Georgia) – Ceyhan(Turkey) (BTC) oil pipeline route-will be dedicated pipeline system to transport up oil from an expanded Sangachal terminal near Baku in Azerbaijan, through Georgia to the Georgian-Turkish border for on ward distribution to Turkish domestic customers via the Ceyhan new terminal. BTC oil pipeline will transport up 50 million ton fresh oil from the Caspian Sea to Ceyhan on the Mediterranean coast in Turkey [13-15];
    - The South Caucasus pipeline (SCP) route-will convey gas by pipeline from Sangachal to the Georgian-Turkish border for on ward distribution to Turkish domestic customers. The proposed SCP pipeline will be 690 km long. In Georgia the SCP new gas pipeline will be constructed parallel to the BTC oil pipeline and it will export 7.3 billion cubic meters (bcm) per year of gas [15].

According to data of European transit countries besides of great political and economical benefits the transit of strategic materials causes great losses to the ecological situation in these countries. Besides of the ordinary pollution of environment there can arise non-ordinary situations as well – accidents on pipes, depositories, which are followed by the sharp deterioration of the ecological situation in the neighboring regions. Thus investigation and assessment of environmental pollution along the TRACEKA route is one of the urgent problems of the present days.

Now we will try to present a short review on environmental baseline for the territory of Georgia as an example of WEP pipeline, TBB railway route and BTC pipeline route.

Western Export Pipeline Route

As known, the WEP fulfils a transportation of oil by pipeline from the expanded Sangachal terminal near Baku in Azerbaijan, through Georgia to Supsa terminal, which is located between Poti and Batumi. WEP pipeline almost fully was blasted by terrorist attacks and sabotage in 1991-1992. Since 1998 to 2000 80% of Baku-Supsa WEP pipeline was restored and 20% – was constructed all over again.

Incidents at Baku-Batumi Railway Route

Oil of the Tengiz deposit (Kazakhstan) is conveyed by railway transport. The route of transportation passes through the territory of Azerbaijan and Georgia to Batumi, where oil is being loaded in tankers and distributed to various countries of Europe.

The length of the Georgian section of railroad is about 385 kilometer. During a day on the territory of Georgia pass about 4 oil railway transports. Each of them transfer (transporting) about 3 500 000 kg of oil (i.e. every day Batumi receives about 4 000 tons of oil) [13].

Transportation of oil of the Tengiz deposit on the territory of Georgia is carried out by the tank-wagons with volume 60 000 kg. There are 6 rail junction station. (Gardabani, Tbilisi, Gori, khashuri, Shorapani, Batumi), 6 tunnels and 34 bridges of the first category) on the rail route of Georgia.

The volume of oil transported by railway is shown in table 1.

Table 1

Year	Volume of transportation in million ton
1998	2.5
1999	3.0
2000	3.5
2001	5.0
2002	5.0
2003	5.0
2004	5.5

How it seems from the Table 1.
The volume of the transporting oil increased from 1998 still 2004 [13].

According to official data of the Georgian Department of Railway Road even in peaceful period since 1979 to 2000 there were 6 big accidents connected with the transportation of oil by the railway. For example in June of 1979 year at the station “Natanebi” – there was collision of the passenger train and the loaded oil train. A lot of people were killed. 90 tons of oil was spilled. About 6 hectare of area was destroyed by fire. Results of the disaster were liquidated after 6 days. In May of 1991 as a result of flood the river Natanebi washed out the pier of the bridge, that destroyed the bridge at moment of passing of the oil train (The bridge "Natanebi" was new – it was built in the late eighties). There was polluted not only basin of the river Natanebi, but also the coast of the Black Sea.

In January of 1996 in the area of the station Tsikhisdziri the oil train ran off the rails. Two tank wagons were turned down, 110 tons of oil was spilled on the slope of the coast of the Black Sea and was spread over the Sea. In April of 2000 in the area of Kharagauli-Marelisi the last three tank wagons ran off the rails. Two of them turned upside-down and 120 tons of oil was spilled in the river Chkerimela. Beside the above cases, there were about 11 small accidents and spilling of the oil in the Black Sea [13-15].

Table 2: Official Data of the Accidents and Spilling of the Oil in the Black Sea from 1996 to 1999

#	Data	Time	Quantity of oil	Area of oil spreading M^2
1	25.02.96	18^{00}	502 kg	600
2	27.02.96	16^{30}	500 kg	600
3	05.09.96	12^{00}	50 kg	
4	23.07.97	15^{00}	20 kg	
5	08.08.98	18^{30}	150 kg	105
6	27.10.98		6 024 kg	1500
7	31.08.99	13^{00}	807 kg	
8	08.07.99	11^{15}	3 230 kg	15 000
9	28.07.98	15^{15}	35 kg	5 000
10	30.08.99	18^{00}		1 800
11	01.12.99		3 kg	

Data on accidents, received from the Georgian Railway Department, allows us to make some statistical characteristics of the space-time distribution of these phenomena. On the Figure 1 is illustrated a probability of the disaster (catastrophes) depending on circumstances (on the axis Ox M is a quantity of spilled oil) [13].

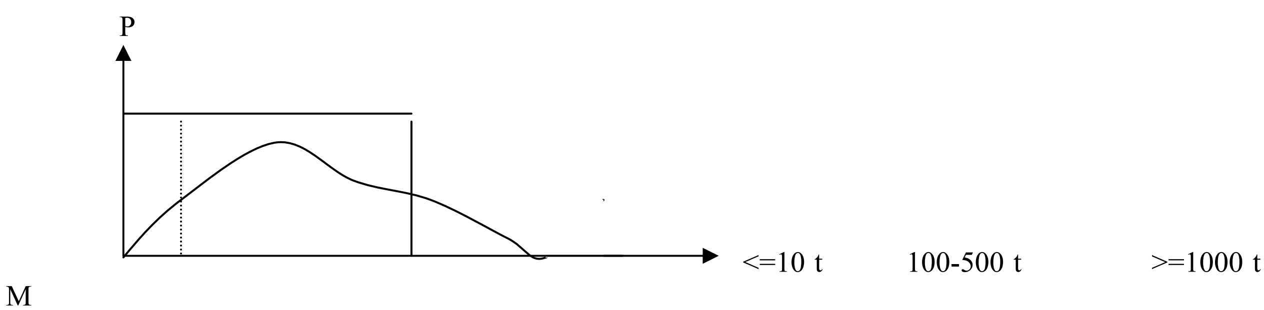

Figure 8.

It can be seen on Fig 8, that an area of small accidents corresponding to 10 tons and area of large disaster corresponding to more than 1000 tons are almost equal. But the area of middle statistical crashes corresponds to 100-500 tons of spilled oil and its probability is higher than small and large accidents.

Now consider Georgian section of Baku-Tbilisi-Ceyhan pipeline route environmental baseline. Gas and oil transportation by pipeline from the Caspian Sea to Turkey via Azerbaijan and Georgia was defined as the most acceptable commercial and environmental solution. Pipeline is generally considered to be most cost effective environmentally safest of transporting hydrocarbons and the route through Georgia was found to be commercially competitive. BTC oil pipeline will transport up 50 million ton fresh oil from the Caspian Sea to Ceyhan on the Mediterranean coast in Turkey. The proposed SCP pipeline will be 690 km long. In Georgia the SCP new gas pipeline will be constructed parallel to the BTC oil pipeline and it will export 7.3 billion cubic meters (bcm) per year of gas [15].

Pipeline trenches will be excavated to a nominal depth of 2.2m. This will vary according to the severity of the terrain and local topography in order to ensure, that the pipeline is buried with a minimum depth of cover of 1m in soil and 0.6m in rock. Deeper installation will be required at river, road, rail and other crossings [15]. But there are able to take place non ordinary situations too.

As foreign practice of pipeline exploitation shows, that the main reason of crashes and spillages (and fires as a consequence) is destruction of pipes as a result of corrosion, defects of welding, natural phenomena and so on (including terrorist attacks and sabotage).

The probability of crashes for pipeline transport rises with the age of the oil pipelines in service, and with the extent of their network. For example, in the US pipeline network with a total length of about 250,000 km occurred 250 ruptures, which are accompanied by spillages of the transferred products, every year in 1973-1983.

In West Europe it has been found, that 10 – 15 leakages occur every year in a pipeline network of around 16,000 km length resulting in a loss of 0.001% of transferred products. According to the project of the company “Kvaerner John Brown”, the system of finding spillages suggested by them allows to determine the location of big spillages in five minutes, and of small ones in two hours. This means that in case of heavy spillages (pipeline break) even in the first five minutes the loss reaches about 64 tons of crude oil. (According to the project the capacity of the pipeline was 762 m^3/h).

Rivers and Under Ground Water

The environmental baseline findings are summarized below. The proposed pipeline route is characterized by very diverse ecological conditions and by abundant biodiversity. The main areas of ecological interest along the route are followings [15]:

1. Tetritskaro outskirts and forest;
2. Bedeni plateau with high mountain meadows of high conservation value and supporting abundant floral biodiversity;
3. Mta taukvetili, that provides a habitat for the Georgian endemic species of blank grouse;
4. Narianis Veli and Kisia wetlands supporting rare high mountain wetland species and providing a habitat for migratory and breeding birds;
5. Tkhratskaro and Kodiana, providing a forest and alpine meadows habitat for large mammals and avian fauna;
6. Tsikhisjvari and Sakire forests with rare floral species and providing a habitat for mammals, amphibians, birds and other faunal groups including rare and endangered species;
7. Mtkvari and Potshuovi rivers crossing with remnants of Reparian forest, supporting high floral and faunal biodiversity.

The proposed pipeline route is generally characterized by arid conditions. The route crosses a multitude of minor watercourses with broad seasonal variations of surface water flow. Six major river crossing occur along the route on the territory of Georgia [15].

It is necessary to carry out additional protective measures for the places, where the oil-pipeline crosses the rivers. Namely, under the rivers the BTC pipeline will be buried deeper in the soil (3-4 meters instead of nominal 2,2 meters) [15]. But the mountain rivers are characterized by the periodical floods. Namely, in Georgia there are frequent cases, when flooded river undermines the bridges, bases and sometimes even take away the bridges. So eventually it can happen the erosion of the soil, decreasing to the minimum of the protective layer of the soil, damaging of the protective coverage of the pipeline by the broken stones brought by the flooded river, development of the corrosion processes and leakage of the oil. In addition, if we take into account, that Georgia is placed in the seismically active zone, it increases the possibility of getting of the spilled oil in the rivers. We think that it is necessary to carry out the control of the water quality and to elaborate such methods and algorithms, which will give us an opportunity to find in time the oil spilled in the river and to identify whether it is leaked from the pipeline or is got into the river as a result of spilling into the water of production waste or occasional spilling.

So that it is necessary: to design a new high-quality river pollution models; to develop new algorithms and means of the control and detection of emergency places on pipelines according to polluted rivers crossing the way of these pipelines; to develop the automated monitoring system of pollution of the river in area of the pipeline and identify a source of its pollution (we have some experience to do it [6-8]).

Ground water along the route is also abundant and generally of high quality. The eastern part of the proposed route is characterized by a shallow water table and by localized poor quality, owing to either high salinity, biological and chemical contamination. The central part of the route is characterized by drinking water used by the local population as the main source of water supply. The western part of the route is characterized by low permeability rocus that locally overlay pressurised mineral water aquifers, including the famous therapeutic water associated with the Borjomi springs.

Numerical investigation of spreading of spilled oil and oil-products along a BTC pipeline

In connections with construction of the new lines of oil pipelines and increase a rail transportation of oil across the territory of Georgia, there is necessity to develop the calculation methods of diffusion, of the spilled oil ingredients. At the present time only a few works are devoted to the investigation of the mechanisms of a filtration and infiltration of oil and polluting substances for the Georgian territory [16,17]. With the help numerical integration of nonlinear filtration equation of a liquid, we have studied distributions of petroleum and mineral oil to the soils in case of their emergency spilling [18,17]. There are ten types of soils along the BTC pipeline rout for the Georgian section [15].

There are mainly four types of soils which occur more frequently along the BTC pipeline rout for the Georgian section. These four types of soils are follows:

1. meadow-alluvial, sandy and sub-sandy (MAS);
2. meadow-marshy (MM);
3. brown- black, peat (BBP);
4. grey-brown, solt (GBS).

According to [16,17] the process of oil-products spreading in soil can be described by the following equation:

$$\frac{\partial S}{\partial t} + K_{11}\frac{\partial S^n}{\partial z} = D_{11}\frac{\partial^2 S^{n+m+1}}{\partial x^2} + D_{11}\frac{\partial^2 S^{n+m+1}}{\partial y^2} + D_{11}\frac{\partial^2 S^{n+m+1}}{\partial z^2}, \tag{5.1}$$

where $K_{11} = \frac{-K_1}{(\sigma - W_0)^n}\cdot\frac{y_W}{y_{oil}},\ D_{11} = \frac{K_1\alpha_1}{(n+m+1)\gamma(\sigma - W_0)^m},\ S = W - W_0.$ (5.2) where W is a saturation of the soil; t – is time; $K(W)$ is a coefficient of filtration; P is a pressure; $\gamma = g\rho$ is a unit weight of liquid. ρ is density; g is gravitational acceleration; .an axis Oz is directed vertically down.

where K_1 is a coefficient of water filtration; when soil is saturated (experiments verified this dependence at $n = 3.5$); W_0 is a comparative volume related with water; σ is a porosity of the soil; y_W, y_{oil} are kinematic coefficients of viscosity of water and oil, respectively.

Here P_0 is a liquid pressure at full saturation of vapors, i.e. when $W = \sigma$, $m \in [-1,+\infty]$.

Taking into account (5), (3) can be rewritten in the following form

$$D(W) = \frac{K(W)\cdot\alpha_1}{\gamma}\left(\frac{W - W_0}{\sigma - W_0}\right)^m, \qquad (5.3)$$

where $\alpha_1 = \dfrac{P_0 y_W}{(\sigma - W_0) y_{oil}}$,

Now we solve the following practical problem. Suppose a big amount of oil is spilled on the Earth surface which has a cylindrical form with a basis Ω_1 and height H_0. And takes the area $\Omega_1 = \Omega_1 * H_0$.

We are interested in the problem of oil filtration and diffusion in the soil, so we seek for solution of the problem (5.1) in the rectangular parallelepiped $G = \{0 \le x \le l_1, 0 \le y \le l_2, 0 \le z \le l_3\}$, until the following condition is fulfilled

$$H(t) = H_0 - \int_t \iiint_G S dG, \quad H(t) \ge 0. \qquad (5.4)$$

We suppose that axis Ox and Oy is oriented in the way, that the upper boundary coincides with the Earth surface, and axis Oz is directed vertically down. See Fig. 9.

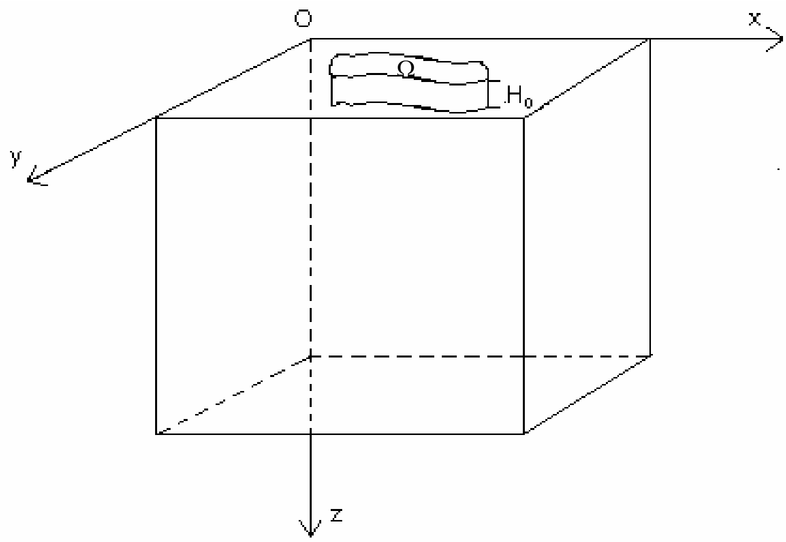

Figure 9.

With the following initial conditions

$$S(O,x,y,z) = -W_0 \text{ at } x,y,z \notin \Omega, \qquad (5.5)$$
$$S(O,x,y,z) = \sigma - W_0 \text{ at } x,y,z \in \Omega,$$

and boundary conditions

$$S(x,y,O,t) = \sigma - W_0 \text{ at } x,y \in \Omega, \qquad (5.6)$$

$$S(x,y,O,t) = -W_0 \text{ at } x,y \notin \Omega, \qquad (5.7)$$

$$\frac{\partial S}{\partial z} = 0, \text{ at } z = l_3 \qquad (5.8)$$

$$\frac{\partial S}{\partial x} = 0, \text{ at } x = l; x = 0; \qquad (5.9)$$

$$\frac{\partial S}{\partial y} = 0, \text{ at } y = 0, y = l_2. \qquad (5.10)$$

In general, problem (5.1), with initial (5.5) and boundary conditions (5,6) – (5.10) is solved numerically.

We have carried out numerical experiments for these four type of soils along BTC pipeline route on the basis of a new filtration equation [16,17]. Numerical integration was carried out during t = 240 days for the MAS soils, t = 265 days for the MM soils, t = 424 days for the BBP soils and t = 270 days for the GBS soils. For an example some of the received results are given in Fig. 10 and 11.

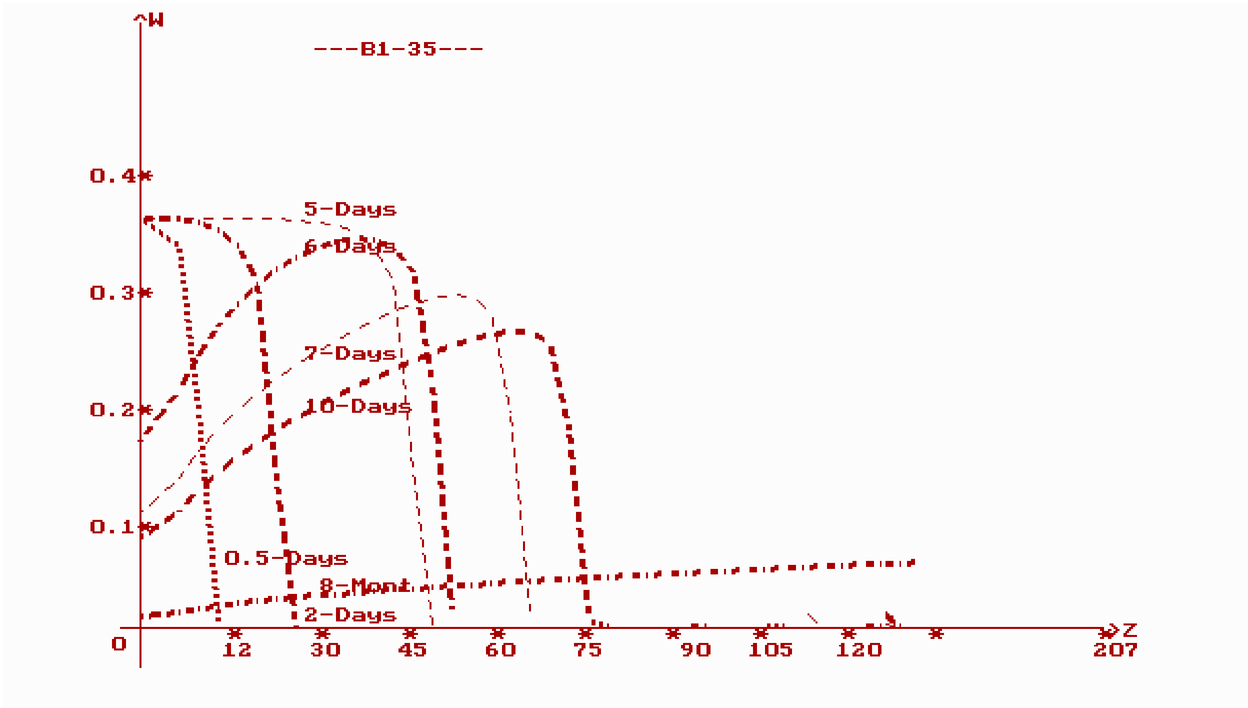

Figure 10: The distribution of the concentrations *W(z)* of the oil at *t = 0.5 2, 5, 7, 10, and 240 days* for the meadow-alluvial, sandy and sub-sandy (MAS) soils.

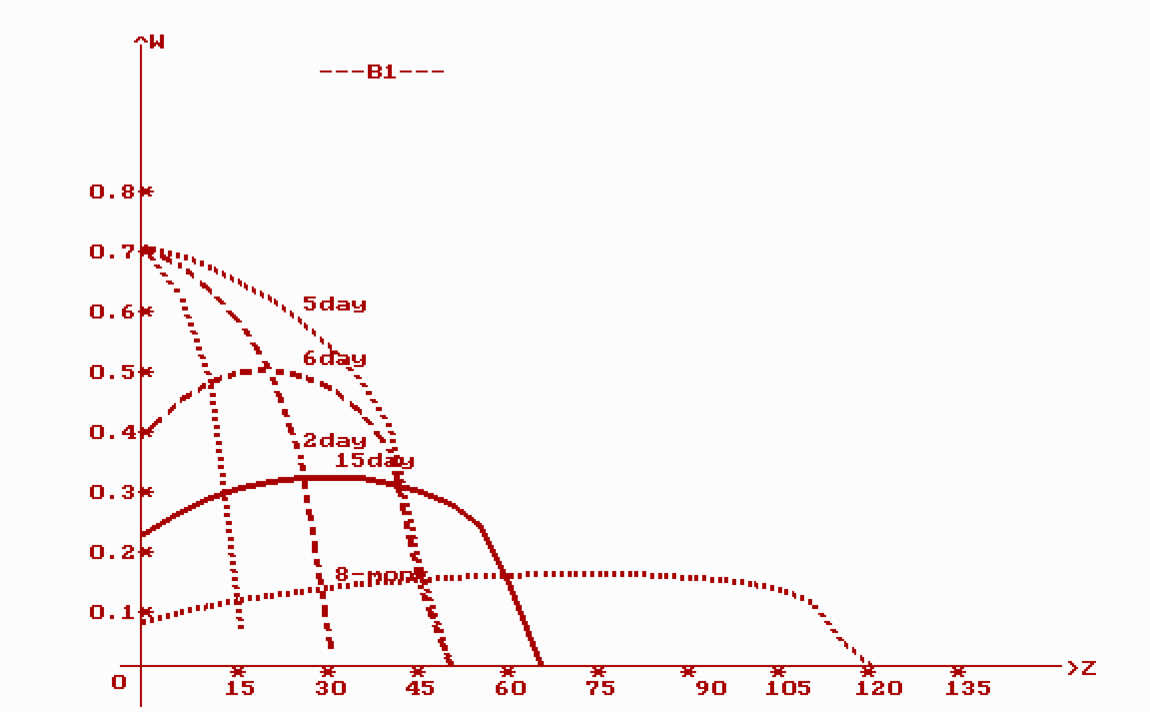

Figure 11: The distribution of the concentrations *W(z)* of the oil at
***t = 0.5 2, 5, 6, 15, and 265 days* for the meadow-marshy soils.**

The numerical calculations showed, that the process of oil infiltration in all considered soils proceeds qualitatively equally i.e. in all considered soils it is possible to distinguished a stage of absorption of the oil in the soil and a stage of distribution of the oil to depth and width of soils. In our case the stage of the oil absorption proceeds about 5 days in the all types of soils (with the exception of the GBS soils when the stage of the oil absorption proceeds about 6.3 days). For this time, an oil spillage with thickness 5cm is fully absorbed by a surface layer of soil. During this stage the value of S (oil concentration) (for the MAS type of soils) is maximal on the soil surface, and it quickly falls with depth i.e. on the depth of 62cm the concentration of oil – S is minimal and extended with width. The front of pollution at $t = 0.5, 2, 5$ and 10 days reaches the depths $z = 9, 27, 48, 75$ cm, respectively. During the second stage of infiltration, concentration of oil-S on a surface of soil gradually decreases (see Fig. 10) [16].

The process of infiltration in the BBP soils is the most intensively and the maximal values of concentration at $t = 5, 10$ days reaches on the depths Z = 26, 50cm respectively. The process of infiltration in the GBS soils is the least intensively. The maximal values of concentration at $t = 5, 10$ days reaches on the depths Z = 12, 21cm respectively. The front of oil pollution at t = 5, 10 days reaches the depths Z = 25, 37cm. The process of infiltration in the MAS soils proceeded qualitatively equally of the BBP soils but this process is less intensive than in BBP soils. The front of oil pollution at t = 5, 10 days reaches the depths Z = 153, 203cm respectively and the maximal values of concentration at $t = 5, 10$ days reaches on the depths Z = 24, 45cm respectively. The process of infiltration in the MM soils proceeded qualitatively equally of the MAS soils but this process is less intensive than in MAS soils. The front of oil pollution at t = 1,2 and 5 days reaches the depths Z = 15, 30 and 48cm respectively and the maximal values of concentration at $t = 15$ and 265 days reaches on the depths Z = 63, 120cm respectively (see Fig. 11).

So we are able to make a brief conclusion that environmental and social-economical baseline of the Georgian section of TRACECA have not studied enough and it is necessary to study this problem more comprehensive. Namely it is necessary to evaluate existing numerical schemes and mathematical models describing oil distribution to environment in case of pipeline damage. They will be used : to study and forecast possible emergency situations, in the air, soils, rivers and under ground water along the TRACECA; to obtain time-space distributions of harmful substances in the air along the TRACECA in the general case and in extra-ordinary situations; to obtain time-space distribution of harmful substances to the soil with account of specific characters of each sort of soils meeting along the TRACECA; to develop the most important task of the river water-monitoring problem – the identification of emergency release sources and the development of operative algorithms and unified computer programs for the common types of water objects and pollution processes; to investigate the impact of pollution on the population in order to prevent harmful contamination and to support a design of sensible economical, social and political abatement strategies.

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